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CWC Mappings and Metrization Theorems

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Abstract: By using the concept of CWC, we establish several new metrization theorems, and obtain the characterization of g-metrizable spaces and symmetric spaces, which generalize some known results.

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It is well known that the concept of g-functions plays an important role in the theory of generalized metric spaces. Lee[4] introduced the concept of CWC mappings as a generalization of g-funtions. In recent years, Gao[1] and Mohamad[10] obtained some metrization theorems by using CWC mappings. It is natural to consider how to characterize some spaces related to weak bases by CWC mappings. Inspired by this idea, we shall give some characterizations of symmetric spaces and g-metrizable spaces.

Also, some new metrization theorems are obtained, which improve some results in [2] and [11].

1 Preliminaries

First, let us recall some definitions.

Definition 1.1[8] Let (X,τ) be a space, and $\mathcal{P}$ a family of subsets of X.

(1) $\mathcal{P}$ is a k-network of X if for every compact subset K and $K \subset U \in \tau$, there exists a finite $\mathcal{P}' \subset \mathcal{P}$ such that $K \subset \bigcup \mathcal{P}' \subset U$.

(2) $\mathcal{P}$ is a weak base of X if $\mathcal{P} = \bigcup_{x\in X} \mathcal{P}_x$ satisfying that (a) For each $x \in X$, $x \in \bigcap \mathcal{P}_x$; (b) If $U, V \in \mathcal{P}_x$, then there exists $W \in \mathcal{P}_x$ such that $W \subset U \bigcap V$; (c) U is an open subset of X if and only if for every $x \in U$, there exists $P \in \mathcal{P}_x$ such that $P \subset U$. The $\mathcal{P}_x$ is called a weak base of x in X for each $x \in X$.

(3) $\mathcal{P}$ is a wcs^*-network of X if for every sequence $\langle x_n\rangle \to x$ and $x \in U \in \tau$, there exist a subsequence $\langle x_{n_k}\rangle$ of $\langle x_n\rangle$ and $P \in \mathcal{P}$ such that $\{x_{n_k}; k \in N\} \subset P \subset U$.

(4) $\mathcal{P}$ is a cs-cover of in X if for every sequence $\langle x_n\rangle \to x$, there exist a $m \in N$ and $P \in \mathcal{P}$ such that $\{x_n; n \geq m\} \subset P$.

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(5) $\mathcal{P}$ is a g-cover of X if every element of $\mathcal{P}$ is a weak neighborhood of some point of X, and for any $x \in X$, there exists some $P \in \mathcal{P}$ such that P is a weak neighborhood of X.

Definition 1.2[8] Let X be a space, a sequence $\{\mathcal{P}_n\}$ of covers of X is a weak development of X if $\{st(x, \mathcal{P}_n) : n \in N\}$ forms a weak base of x in X for each $x \in X$.

A function $g : N \times X \to \tau(X)$ is called g function if $g(n+1, x) \subset g(n, x)$ for any $x \in X$ and $n \in N$.

Definition 1.3[4] Let X be a space, a function $g : \omega \times X \to \mathcal{P}(X)$ is called CWC, if for every $x \in X$ and $n \in \omega$, $x \in g(n+1, x) \subset g(n, x)$, and for every subset U of X, U is open if and only if for each $x \in U$, there exists $n \in \omega$ such that $g(n, x) \subset U$.

g-metrizable spaces are the regular spaces with σ locally finite weak bases. A space X is g-development, if there exists a CWC mapping g on X with the property: if $x, x_n \in g(n, y_n)$ for each $n \in N$, the sequence $\{x_n\}$ converge to x.

Definition 1.4 Let $\mathcal{F}$ be a collection of X. $\mathcal{F}$ is called LF, if for every $x \in X$, there exists a neighborhood U of x such that $| \{K \bigcap F : F \in \mathcal{F}\} | < \omega$.

Let $\langle x_n >$ be a sequence of X, $x_n \to x$. We denote $T(x) = \{x_n : n \in N\} \bigcup \{x\}$.

Throughout this paper, all spaces are assumed to be T_2-spaces.

2 Main Results

Theorem 2.1 For a space X, the following are equivalent:

(1) X is symmetrizable;

(2) X have a weak development;

(3) There exists a CWC mapping g on X satisfying that for any sequence $\langle x_n \rangle$ and point x in X, $x_n \to x$ if $x \in g(n, x_n)$;

(4) There exists a CWC mapping g on X satisfying that for any sequence $\langle x_n \rangle$ and point x in X, if $x \in g(n, x_n)$, then $x_{n_k} \to x$ for some subsequence $\langle x_{n_k} \rangle$ of $\langle x_n \rangle$.

Proof (1) $\Leftrightarrow$ (2) See Proposition 1.6.12[7].

(2) $\Rightarrow$ (3) Suppose $\langle \mathcal{B}_n \rangle$ is a weak development of X, and $\mathcal{B}_{n+1} < \mathcal{B}_n$. Let $g(n, x) = st(x, \mathcal{B}_n)$, then g is a CWC mapping. If $x \in g(n, x_n)$, then $x \in st(x_n, \mathcal{B}_n)$, thus $x_n \in st(x, \mathcal{B}_n)$, so $x_n \to x$.

(3) $\Rightarrow$ (4) Obviously.

(4) $\Rightarrow$ (1) Let g be a CWC mapping satisfying (4), and $m(x, y) = \min\{n, x \notin g(n, y), y \notin g(n, x)\}$, then $m(x, y) = m(y, x) \in N$ whenever $x \neq y$.

For each $x, y \in X$, let $d(x, y) = \frac{1}{m(x,y)}$ if $x \neq y$, $d(x, y) = 0$ if $x = y$. We show that d is a symmetric.

(i) Let $x \in U \in \tau$, then there exists $n_0 \in N$ such that $g(n_0, x) \subset U$. If not, we can pick $x_n \in g(n, x) - U$, then $x_{n_k} \to x$, a contradiction. On the other hand, there exists $n \in N$ such that $B(x, \frac{1}{n}) \subset g(n_0, x)$. If not, let $y_n \in B(x, \frac{1}{n}) - g(n_0, x)$, since $y_n \in B(x, \frac{1}{n})$, then $m(x, y_n) > n$, hence $x \in g(n, y_n)$ whenever $n > n_0$, so there exists a subsequence y_{n_k} of y_n converging to x, which contracts $y_n \notin g(n_0, x)$.

(ii) For every subset U of X, if for each $x \in U$, there exists $n \in N$ such that $B(x, \frac{1}{n}) \subset U$, then U is open.

In fact, it is easy to see that $g(n,x) \subset B(x, \frac{1}{n})$, so (ii) is true.

By (i) and (ii), d is symmetric.

Theorem 2.2 For a regular space X, the following are equivalent:

(1) X is metrizable;

(2) There exists a CWC mapping g on X satisfying that for any closed set F and compact set C in X, $(\bigcup_{x\in F} g(n,x)) \bigcap (\bigcup_{x\in C} g(n,x)) = \emptyset$ if $F \bigcap C = \emptyset$;

(3) There exists a CWC mapping g on X satisfying that for the sequences $\langle x_n \rangle$ and $\langle y_n \rangle$ of X, if $x_n \to x$ and $g(n,x_n) \bigcap g(n,y_n) \neq \emptyset$ for each $n \in \omega$, then $y_n \to x$.

(4) X has a weak development $\langle \mathcal{P}_n \rangle$ such that for every sequence $x_n \to x$ and $T(x) \subset U$, there exists $n \in N$ such that $st(T(x), \mathcal{P}_n) \subset U$.

(5) There exists a CWC mapping g on X satisfying

(I) For the sequences $\langle x_n \rangle$ and $\langle y_n \rangle$ of X, if $x_n \to x$, and $y_n \in g(n, x_n)$, then $y_n \to x$;

(II) For the sequences $\langle x_n \rangle$ and $\langle y_n \rangle$ of X, if $x_n \to x$, and $x_n \in g(n, y_n)$, then $y_n \to x$.

Proof By Theorem 1 in [6], (1) $\Leftrightarrow$ (4). It is easy to see (1) $\Rightarrow$ (2) and (3) $\Rightarrow$ (5), we only need prove (2) $\Rightarrow$ (3) and (5) $\Rightarrow$ (4).

(2) $\Rightarrow$ (3). Let g be a CWC mapping satisfying (2), and x_n, y_n be sequences satisfying the conditions in (3). If $y_n \not\to x$, then there exists an open neighborhood V of x and subsequence sequence y_{n_k} such that $\{y_{n_k} : k \in N\} \subset X - V$. Let $F = X - V$ and $C = \{x_n : n \in N; x_n \in V\} \bigcup \{x\}$, then F and C are closed set and compact set respectively, and $F \bigcap C = \emptyset$. By (2), there exists $n \in N$ such that $(\bigcup_{y\in F} g(n,y)) \bigcap (\bigcup_{z\in C} g(n,z)) = \emptyset$, thus we can pick $k \in \omega$ such that $g(n_k, x_{n_k}) \bigcap g(n_k, y_{n_k}) = \emptyset$, a contradiction.

(5) $\Rightarrow$ (4).

Let g be a CWBC mapping satisfying (5). For every $x, y \in X$, let $d(x,y) = \frac{1}{m(x,y)}$ if $x \neq y$, $d(x,y) = 0$ if $x = y$ where $m(x,y) = \min\{n, x \notin g(n,y), y \notin g(n,x)\}$, then d is a symmetric function on X.

Let $\mathcal{U}_n = \{A, \mathrm{diam} A < \frac{1}{n}, A \subset X\}$, then $st(x, \mathcal{U}_n) = B(x, \frac{1}{n})$, so $\langle \mathcal{U}_n \rangle$ is a weak development of X. Let $T(x) = \{x_n, n \in N\} \bigcup \{x\}$, and $T(x) \subset U \in \tau(X)$, we show that there exists $n \in N$ such that $st(T(x), \mathcal{P}_n) \subset U$.

If not, for each $n \in \omega$, $st(T(x), \mathcal{P}_n) \not\subset U$. We can obtain a subsequence $\langle x_{n_k} \rangle$ of $\langle x_n \rangle$ such that $st(x_{n_k}, \mathcal{P}_k) - U \neq \emptyset$. For each $k \in \omega$, Pick $y_k \in st(x_{n_k}, \mathcal{P}_k) - U$, then $d(y_k, x_{n_k}) < \frac{1}{k}$, namely, $m(y_k, x_{n_k}) > k$, so $x_{n_k} \in g(k, y_k)$ or $y_k \in g(k, x_{n_k})$.

There exists a subsequence $\langle x_{n_{k_j}} \rangle$ of $\langle x_{n_k} \rangle$ satisfying $x_{n_{k_j}} \in g(k_j, y_{k_j})$ or a subsequence $\langle y_{k_j} \rangle$ of $\langle y_k \rangle$ satisfying $y_{k_j} \in g(k, x_{n_{k_j}})$, thus $y_{k_j} \to x$ by (I) or (II), a contradiction.

The following corollary improves a result in Top. Appl. by C. Good[2].

Corollary 2.3 A space X is metrizable if and only if X has a CWC mapping g satisfying that for any compact set C and closed set F, $C \bigcap F = \emptyset$, then there is some $n \in N$ such that for all $x \in X$, $g(n,x)$ meets at most one of C and D.

Proof We only need prove the sufficiency. Let g be the mapping as above. We show that g satisfies (I) and (II) in Theorem 2.2. Let $\langle x_n\rangle, \langle y_n\rangle$ be sequences of X, and $x_n \to x$, and $y_n \in g(n, x_n)$. If $\langle y_n\rangle \not\to x$, then there exists an open neighborhood V of x and subsequence sequence y_{n_k} such that $\{y_{n_k} : k \in N\} \subset X - V$. Let $F = X - V$ and $C = \{x_n : n \in N; x_n \in V\} \bigcup \{x\}$, then F and C are closed set and compact set respectively, and $F \bigcap C = \emptyset$. There is some $m \in N$ such that for all $x \in X$, $g(m, x)$ meets at most one of C and D, but $g(n_m, x_{n_m}) \bigcap F \neq \emptyset$, and $g(n_m, x_{n_m}) \bigcap C \neq \emptyset$, a contradiction, so (I) is true. The proof of (II) is similarly.

Corollary 2.4[11] For a space X, the following are equivalent:

(1) X is metrizable;

(2) There exists a g function on X satisfying that for any closed set F and compact C in X, $(\bigcup_{x\in F} g(n,x)) \bigcap (\bigcup_{x\in C} g(n,x)) = \emptyset$ if $F \bigcap C = \emptyset$;

(3) There exists a g function on X satisfying that for the sequences $\langle x_n\rangle$ and $\langle y_n\rangle$ of X, if $x_n \to x$, and $g(n, x_n) \bigcap g(n, y_n) \neq \emptyset$ for each $n \in N$, then $y_n \to x$.

Proof It is not difficult to see that every g function in (3) is a CWC mapping, so we only need prove that X is regular if there exists a g function on X satisfying the conditions in (3). In fact, Let $x \notin F, F$ is a closed of X, then there exists $m \in N$ such that $g(m, x) \bigcap g(m, F) = \emptyset$, thus X is regular. If not, for each $n \in N$, we can pick $y_n \in F$ such that $g(n, x) \bigcap g(n, y_n) \neq \emptyset$, so $y_n \to x$, a contradiction.

LF property of g functions was used to characterize metrizable spaces, Lasnev spaces and $\aleph$ spaces in [12]. From Theorem 12 and Corollary 4 in [1], it is easy to see that the Theorem 2.5 is true.

Theorem 2.5 For a regular space X, the following are equivalent:

(1) X is metrizable;

(2) There exists a CWC mapping g on X satisfying that

(I) For the sequences $\langle x_n\rangle$ and $\langle y_n\rangle$ of X, if $x_n \to x$, and $y_n \in g(n, x_n)$, then $y_n \to x$;

(II′) For the sequence $\langle x_n\rangle$ of X, if $x \in \overline{g(n, x_n)}$, then $x_n \to x$;

(III) For each $n \in N$, $\{g(n, x), x \in X\}$ is LF.

The next theorem shows differences between metrizable spaces and g-metrizable spaces by CWC mappings.

Theorem 2.6 A regular space X is g-metrizable if and only if there exists a CWC mapping g on X satisfying (I) and (III).

Proof Necessity: Let $\mathcal{B} = \bigcup \mathcal{B}_n$ be a weak base of X where $\mathcal{B}_n$ is a discrete collection of closed subsets of X for each $n \in N$, and $\mathcal{B}_x$ be the weak base of x for each $x \in X$. Let $h(n, x) = P \in \mathcal{P}_n$ if $\mathcal{P}_n \bigcap \mathcal{B}_x \neq \emptyset$, and $h(n, x) = X - \bigcup\{P : P \in \mathcal{B}_n, x \notin P\}$ if $\mathcal{P}_n \bigcap \mathcal{B}_x = \emptyset$. We show that $\{g(n, x) : x \in X\}$ is LF for each $n \in N$. In fact, for every $x \in X$ and $n \in N$, there exists a neighborhood U such that U meets at most one element of $\mathcal{B}_n$, then $\{U \bigcap h(n, x), x \in X\} \subset \{U, U - P, U \bigcap P; P \bigcap U \neq \emptyset\}$, so $\{h(n, x) : x \in X\}$ is LF.

Let $g(n, x) = \bigcap\{h(k, x); 1 \leq k \leq n\}$, then $g(n, x)$ is LF for each $n \in N$. We only need prove $g(n, x)$ satisfying (I). It is not difficult to see that if $z \in g(n, y)$, and $z \in P \in \mathcal{P}_n$, then $y \in P$.

Let $x_n \in g(n, y_n)$, $x_n \to x$. For each $P \in \mathcal{B}_x \bigcap \mathcal{B}_n$, there exists some $m \in N$ such that $x_k \in P$ whenever $k \geq m$, thus $y_k \in P$, so $y_n \to x$.

Sufficiency: For each $n \in N$ and $x \in X$, let $h(n,x) = \bigcap\{g(n,y) : x \in g(n,y)\} \setminus (\bigcup\{g(n,y) : x \notin g(n,y)\})$. Then $x \in h(n,x) \subset g(n,x)$. Let $\mathcal{H}_n = \{h(n,x) : x \in X\}$. Then $\mathcal{H}_n$ is a locally-finite cover of X. Let $x \in X$ and let U be a neighborhood of x such that $|\{U \cap g(n,y) : y \in X\}| < \omega$. Then $|\{U \cap h(n,y) : y \in X\}| < \omega$. Since $\mathcal{H}_n$ is a partition of X, U can meet only with finitely many $h(n,y)$, and hence $\mathcal{H}_n$ is locally-finite.

Now, let $\mathcal{H} = \bigcup_{n \in N} H_n$. We shall show that $\mathcal{H}$ is a *wcs*-network of X. For each $x \in X$ and U be a neighborhood of x and let $x_i \to x$. Then there exist $n_0, i_0 \in N$ such that $st(x_i, \mathcal{H}_{n_0}) \subset U$ for each $i > i_0$. Otherwise, for each $n \in N$ we can find $i_n > n$ such that $st(x_{i_n}, \mathcal{H}_n) \setminus U \neq \emptyset$. Pick $y_n \in st(x_{i_n}, \mathcal{H}_n) \setminus U$. Then for each $n \in N$, there exists z_n such that $x_{i_n}, y_n \in h(n, z_n)$. Since $x_{i_n} \to x$ and $x_{i_n} \in h(n, z_n) \subset g(n, z_n)$, $z_n \to x$. But it follows from $y_n \in h(n, z_n)$ that $z_n \in g(n, y_n)$, and hence $y_n \to x$, which contradicts that $y_n \notin U$ for each $n \in N$.

Since $\mathcal{H}_{n_0}$ is a locally-finite cover of X, $\{x_i : i > i_0\}$ can only meet with finitely many members of $\mathcal{H}_{n_0}$ and we can take $H_1, H_2, \cdots, H_k \in \mathcal{H}_{n_0}$ such that $\{x_i : i > i_0\} \subset \bigcup\{H_j : j \leq k\}$ and $H_j \cap \{x_i : i > i_0\} \neq \emptyset$ for each $j \leq k$. By the last condition, we have $\bigcup\{H_j : j \leq k\} \subset \{st(x_i, \mathcal{H}_{n_0}) : i > i_0\} \subset U$. So $\mathcal{H}$ is a *wcs*-network. Hence, X is a g-metrizable space.

Finally, we discuss the relations between g-metrizable spaces and the space with a weak development of locally finite *cs*-covers. It is not difficult to see that the following is true for a regular space X:

Metric spaces ⇒ Spaces having a weak development of locally-finite *cs*-covers ⇒ g-metrizable spaces. We shall show that the converse is not hold.

Theorem 2.7 Spaces having a weak development of point-finite *cs*-covers are g-developable.

Proof Let X be a spaces having a weak development $\mathcal{U}_n$ such that each $\mathcal{U}_n$ is a point-finite *cs*-cover. For each $n \in N$, let $\mathcal{G}_n = \bigwedge_{i \leq n} \mathcal{U}_i$. Then $\mathcal{G}_n$ is a point-finite *cs*-cover of X. By Lemma 3 in [5], we can assume that each $\mathcal{G}_n$ is a g-cover of X. For each $n \in N$ and $x \in X$, take $g(n,x) \in \mathcal{G}_n$ such that g is a CWC-map on X. If $x, x_n \in g(n, y_n)$ for each $n \in N$, then $x_n \in st(x, \mathcal{G}_n)$, so the sequence $\{x_n\}$ converge to x, hence X is g-developable.

It is easy to check that a space has a weak development of point-finite(resp. locally-finite) g-covers if and only if it has a point-star network of point-finite(resp. locally-finite) g-covers.

L. Foged in [3] constructed an counterexample showing g-metrizable can not be g-developable, which shows that g-metrizable spaces can not be spaces having a weak development of point-finite *cs*-covers by Theorem 2.

Example 2.8 S_2 has a weak development of locally-finite *cs*-covers.

Let $S_2 = \{0\} \cup N \cup (N \times N)$ with the Arens' topology. For each $n, i \in N$, let $F_n = \{1, 2, \cdots, n\}, V(i,n) = \{i\} \cup \{(i,k) \in N \times N : k \geq n\}, W(n) = \bigcup_{i \in N \setminus F_n} V(i,n), \mathcal{G}_n = \{\{0\} \cup (N \setminus F_n)\} \cup \{V(i,n) : i \in F_n\} \cup \{W(n)\} \cup \{\{x\}\} : x \in N \times N\}$. Then $\mathcal{G}_n$ is a weak development with each $\mathcal{G}_n$ a locally-finite *cs*-cover of S_2.

Theorem 2.9 Spaces having a weak development of locally-finite g-covers are a 1-sequence-covering, compact-covering, quotient, compact and σ-image of a metric space.

Proof Let $\mathcal{P}_i$ be a weak development of locally-finite g-covers of a space X. And let $(f, M, X, \mathcal{P}_i)$ be a Ponomarev's system. Then f is a compact-covering, quotient, compact and σ-mapping by Theorem 14 in [9], and f is a 1-sequence-covering mapping by Theorem 3.1.7 in [8].

Question 2.10 (1) Is the converse of Theorem 3 hold?

(2) Is g-metrizable and g-developable space a space having a weak development of locally-finite cs-covers?

References

[1] Gao Z.M., , Applications of the weak base g-functions in metrizability of spaces, *Advances in Mathematics(China)*, 2003, 32(6): 653-659.

[2] Chris Good, Daniel Jennings, Abdul M. Mohamad, symmetric g-functions, *Top. Appl.*, 2003, 134: 111-122.

[3] Foged, L., On g-metrizability, *Pacific J. Math.*, 1982, 98(2): 327-332.

[4] Lee, K.B., On certain g-first countable spaces, *Pacific J. Math.*, 1976, 65(1): 113-118.

[5] Li Jinjin, Li Kedian, The sequence-covering compact images of paracompact locally compact spaces, *Northeast. Math. J.*, 2000, 16(4): 463-468.

[6] Lin Shou, Weak base and metrization theorems, *J. Sichuan University*, 1993, 30(2): 164-166.

[7] Lin Shou, Generalized Metric Spaces and Mappings, Beijing: Chinese Science Publishers, 1995(in Chinese)

[8] Lin Shou, Point-countable Covers and Sequence-covering Mappings, Beijing, Chinese Science Press, 2002.

[9] Lin Shou, Yan Penglei, Notes on cfp-covers, CMUC, 2003, 44(2): 295-306.

[10] Mohamad, A.M., Conditions which imply metrizability in some generalized metric spaces, *Top. Proc.*(spring) 1999, 24: 215-232.

[11] Peng Liangxue, Lin Shou, On monotone spaces and ametrization theorems, *Acta Math. Sinica(in Chinese)*, 2003, 46(6): 1225-1232.

[12] Yun Z.Q., Yang X.H. and Ge Y., Metrizable, ℵ-Lasnev spaces and g-functions. *Math. Japonica*, 2000, 53(2): 225-229.

CWC 映射和度量化定理

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摘要： 利用 CWC 映射，本文获得了对称度量空间和 g 可度量空间的特征，建立了几个度量化定理，改进了一些已知结果. 主要的定理是证明正则空间 X 是可度量化空间当且仅当存在 X 上的 CWC 映射 g 满足如下条件：

(I) 若序列 $\{x_n\}$ 和 $\{y_n\}$ 对于每一 $n \in N$ 有 $x_n \in g(n, y_n)$ 且 $x_n \to x$，则 $y_n \to x$.

(II) 若序列 $\{x_n\}$ 和 $\{y_n\}$ 对于每一 $n \in N$ 有 $y_n \in g(n, y_n)$ 且 $x_n \to x$，则 $y_n \to x$.

关键词： 可度量化空间； CWC 映射； 对称度量空间； g 可度量空间